

## FRAMEWORK FOR THE EVALUATION OF PROFICIENCY – BASED MATHEMATICS INSTRUCTIONAL MATERIALS

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### **Abstract**

The multifarious role play by mathematics instructional materials in teaching and learning is evident and the process of adoption of textbooks for students is critical to the selection of high quality instructional materials. Hence, the instructional material in the textbooks is one that must be scrutinized through comprehensive and reliable procedure and must be supported by published research in the field of mathematics education. The selected curriculum materials must align with and support the need for implementation of state or country level curriculum standards. The purpose of this paper is to provide a comprehensive research based framework that analyzes salient features of instructional materials which builds mathematical proficiency among students of middle grades. The framework will assist state level text books evaluation teams, school administrators, and teachers in selecting mathematics curriculum materials that support implementation of mathematics curriculum standards. The proposed framework may also provide significant information that can be useful for curriculum developers and schools while they are making decisions regarding modifications in available mathematics materials and monitoring the quality of published materials for better attainment of students' learning.

### **Keywords:**

Curriculum standards  
Evaluation framework  
Instructional materials  
Mathematical proficiency

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## 1. Introduction

The major apprehension in all standards-based curriculum reforms is the inclusion of knowledge, understanding, and skills that constitute a competency in a field. Thus, the first characteristic of standards-based materials is the focus on core Mathematics for all students. Despite many difficulties, current studies provide evidence of the positive impact of Standards-based curriculum materials. Robert Reys et al. (2003) conducted research to measure the impact of standards-based Mathematics text books on students of eighth graders. He compared the Mathematics achievement of students who were using standard based curriculum materials with the students using other curriculum materials. The research identified significant differences in the achievements between the students using the standards-based curriculum materials and students using other curriculum materials. Using Standards-based materials in content standard areas, students scored higher in all the three districts. Similarly, research conducted by Fuson, Carroll, and Drucek (2000) found that students learning from Everyday Mathematics (EM), an elementary Standards-based curriculum, scored as well as or better than students studying from traditional materials on standard topics including place value and computation. Additionally, the Standards-based group got the capability to study a varied choice of curriculum topics (e.g., geometry, fractions, and algebra) usually not given appropriate consideration in traditional materials. Also, it was identified that the opportunity led to enhance learning. For instance, the EM group considerably well performed in the National Assessment of Educational Progress sample on geometry items (Fuson et al, 2000).

Instructional materials have a particularly important role in making changes happen, the processes students' use, the way teachers teach, and what is assessed (William H et al. 1997:2). The textbook Publishers use curriculum standards to design textbooks and other classroom instructional materials to implement the intended curriculum. These materials include textbooks typically developed to support the daily teaching of mathematics in classroom. The role of the textbook varies greatly from context to context and teacher to teacher. Some researchers found that Mathematics teachers depend on textbooks for at least 90% of their classroom teaching time (Mikk, 2000). Such evidences demand the need for quality textbooks. While Sewall (1992) say that without the use of textbooks attainment of education is almost impossible. Valverde et al., (2002) regard the textbook as the potentially implemented curriculum. He created a connection

between the intended and the implemented curricula in their creation of the potentially implemented curriculum, affected primarily by the textbook.

There are increasing apprehensions about students' achievement in mathematics which is evident from the studies of international researchers (Schmidt, McKnight, & Raizen, 1997; Wu, 1997). There is strong agreement among mathematics researchers that the quality instructional materials be improved to enhance the teaching and learning of mathematics among teachers and students. Modifications have been made in curriculum documents; especially in previous couple of decades, to improve practice in mathematics classrooms and introduction of inquiry based teaching approaches have led to concerns regarding the quality of mathematics textbooks.

In the 1990s, the National Council of Teachers of Mathematics (NCTM) published its Standards documents (e.g., NCTM, 1989, 1991, 1995), which proposed recommendations for reformation and renovation of K-12 school mathematics. Plenty of school mathematics materials were developed and applied to align with the recommendations of the NCTM Standards and these Standards-based mathematics curricula has also impacted significantly on students' learning in Mathematics (e.g., Schoenfeld, 2006; Senk & Thompson, 2003). As the standards-based curricula, develops students' proficiency in mathematics through investigations of real-world situations and problems. At the same time many publishers of so-called traditional curricula also claim about their textbooks and other material being Standards-based. So these questions may arise in minds. How salient features of standards-based instructional materials be analyzed to distinguish them from traditional materials? This question talk about the development and analysis of the learning goals and which is the major work of current research.

The proposed framework for the evaluation of instructional materials provides an authentic procedure and look for salient features of instructional materials which builds proficiency among students and teachers in learning mathematics. By Instructional/curriculum "materials" here means those resources in the form of text books or other materials in printed or soft form that are used as daily guides for students or used by teachers in leading classroom activities related to mathematics teaching and learning. It is also expected that by involving in the evaluation procedure teacher and decision makers will broaden the knowledge of how to evaluate important

features of curriculum materials. This instructional material evaluation framework can support the development and success of evolving curricula and new teaching and learning initiatives.

## **2. Historical overview of Evaluation Frameworks**

Different authors and international organizations have conducted studies on methods of analyzing various aspects of mathematics textbooks. TIMSS developed a curriculum framework, based on tripartite model, to compare systems of education across the nations through analyses of curricula, related documents and artifacts (Robitaille et al., 1997): Intended curriculum, Implemented curriculum and Attained curriculum.

Project 2061 has designed a curriculum materials evaluation process that reliably recognizes mathematics instructional materials whether aligned with specific content standards and reveals effective instructional methods (Roseman, Kesidou, & Stern, 1996; Kulm, Morris & Grier 2000). Project 2061 procedure for analyzing mathematics textbooks attends to both content and instructional design to explain these questions: Does the textbook focus on a coherent set of significant, age-appropriate student learning goals? Whether the material's instructional design efficiently supports the achievement of those specified learning goals? The learning goals and criteria for evaluation were derived from significant documents including the NCTM *Standards* (National Council of Teachers of Mathematics, 1989) and the *Benchmarks for Science Literacy* (American Association for the Advancement of Science, 1994). The evaluation criteria were comprised of seven categories including: Building on student ideas about mathematics; Engaging students in mathematics; Developing mathematical ideas; and Promoting student thinking about mathematics.

Common Core State Standards (CCSS) Mathematics Curriculum Materials Analysis Project provided a set of tools that assist K-12 textbook adoption committees, school administrators, and K-12 teachers in selecting mathematics curriculum materials that support implementation of the newly developed common core state standards of mathematics (CCSSM). The tools are designed to provide educators with objective measures and information to guide their selection of mathematics curriculum materials based on evidence of the materials' alignment with the CCSSM and support for implementation of the CCSSM in classrooms

National Assessment of Educational Progress (NAEP) used a framework in mathematics assessment which features three mathematical abilities (conceptual understanding, procedural knowledge, and problem solving) and includes additional specifications for reasoning, connections, and communication (National Assessment Governing Board, 2000).

### **3. Proposed Framework for Evaluation of Instructional Materials**

The current study is influenced by the work of Kilpatrick, Swafford & Findell (2001). Kilpatrick et al. (2001: 116) used the term Mathematical Proficiency (MP) to capture the knowledge which is necessary for anyone to learn mathematics successfully. Kilpatrick identified five interwoven and interdependent Mathematical proficiency strands: 1) Conceptual understanding, 2) Procedural fluency, 3) Strategic competency, 4) Adaptive reasoning, 5) Productive disposition. Kilpatrick et al. (2001: 115) contended that students will be proficient in mathematics if they are proficient in these five strands.

**Conceptual Understanding:** Comprehension of mathematical concepts, operations, and relations.

**Procedural Fluency:** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

**Strategic Competence:** Ability to formulate, represent, and solve mathematical problems.

**Adaptive reasoning:** Capacity for logical thought, reflection, explanation, and justification.

**Productive disposition:** Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick et al. 2001: 115).

**Assessment:** The author included an important component 'Assessment' which is one of the integral part of teaching and learning and must be the part of instructional materials. It is reasonable to evaluate that how instructional materials incorporate Assessment strategies to monitor teaching and learning. The studies showed that most of the curriculum materials and teacher made standardized tests, focused mainly on algorithmic procedure and did not provide extensive opportunities for learning different kinds of concepts and logical thinking (Bergqvist, 2007; Lithner, 200, 2003, 2004, 2008). Furthermore, there is a relation between task

characteristics, in terms of types of tasks, and the mathematical reasoning students use when solving tasks in a context of a test situation (Boesen, J., Lithner, J., & Palm, T. (2010). Attractive and contextual mathematical task may help students enhance thinking and reasoning skills. The textbooks mediate to transform intended curriculum into teaching materials and teachers widely use textbooks to plan, teach and assess students' learning in the classrooms (Valverde et al., 2002). Also, teachers rely on the assessment strategies incorporated by published textbooks for students to know and be able to do in mathematics (Thompson, Hunsader, & Zorin, 2013). Senk, Beckmann, & Thompson, (1997) conducted a research on US high school teachers' use of assessments and found that over half of the teachers used assessments strategies provided in their textbook's ancillary materials.

These six standards, including Kilpatrick's five mathematics proficiency strands and the Assessment component suggested by the author provide the basis for a comprehensive framework for evaluation of instructional materials in mathematics and which is supported by strong conceptual theory and research literature.

#### **4. Development of Instructional Goals and Indicators.**

As mentioned above, six strands including Kilpatrick's five proficiency strands plus Assessment standard, form the proficiency standards for the proposed framework to evaluate mathematics instructional materials. A set of 9 mathematics instructional goals was developed for these six standards which form the basis for the analysis of instructional materials. Indicators were developed from large materials of relevant published research. For the validation of instructional goals and to assure that the goals for evaluating materials should cover fundamental concepts and skills in middle level mathematics materials, Statements of the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) were used as reference standards. Instructional goals were compared to align with statements in the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000). Also, keen attention was given to the development of indicators which defines specific areas in each goal that might be included in the materials and provide a clear guideline for a deeper understanding of the areas to evaluate instructional materials. A template is given in table 1,

which provides guidance for planning when using the Framework for Instructional Material Evaluation.

## **5. Description and Justification of Instructional Goals and Indicators**

The following is a brief description of goals and its indicators for six proficiency standards supported by research literature.

### **5.1. Conceptual Understanding**

Students demonstrate “*conceptual understanding*” in mathematics when they recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts (NAEP, 2003; *What Does the NAEP Mathematics Assessment Measure?*). Conceptual understanding refers to an integrated and functional grasp of mathematical ideas (Kilpatrick et al. 2001:139). Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They can shape their understanding into a coherent whole, which assists them to understand new concepts by connecting those concepts to their previous ideas, (Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.).1999). A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. To find one’s way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. (Kilpatrick et al. 2001:119).

Connections are most expedient for conceptual understanding of students when they connect interrelated concepts and methods in appropriate ways. The conceptual understanding of students depends upon the level of depth of knowledge and extent of the connections they have made. Thus, conceptual understanding is innately connected to representations. Lesh, Post and Behr (1987) even linked understanding with the capacity to recognize, manipulate and translate an idea/concept in and between different representations, thus this also emphasize the point of

connections. When students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding (Kilpatrick et al. 2001:20).

***Goal 1. The material develops mathematical concepts in students.***

Instructional materials should, provide opportunities for students to make connections between and among mathematical ideas and skills. Students cannot understand concepts by merely building up bits of knowledge unless an explicit support in linking ideas is extended (Resnick, 1987). Other factors which help to maintain students' engagement at a high level includes the type of tasks that build on students' prior knowledge. The arrangement of tasks and activities are inappropriate if the materials do not confront students' prior knowledge and teachers are clearly directed to it (Mack. 1990). The materials should guide teachers in helping students formulate the process themselves, building on their existing knowledge.

The use of scaffolding is another factor that helps to maintain student engagement at a high level (Kilpatrick et al, 2001:371). Scaffolded instruction is one way of structuring explicit instruction. Building on Vygotsky's (1978) theory of the Zone of Proximal Development, Carolan & Guinn, (2007) described scaffolding as a system of temporary supports that are designed to help a learner, bridge the gap between where they are and what they can do, and where they need to be and be able to do in order to be successful with a learning task. Scaffolded practice must be present in every lesson in mathematics textbooks. This instructional goal involves examining the materials, whether they include models or foundation level skills for such kind of practices and whether the materials gradually formalize the skills with terminology and mathematical structure layered onto the skills? Responding to the "prerequisites" criterion involves making a list of prerequisite concepts and/or skills, examining whether the material has sufficiently addressed the prerequisites in the earlier units and examining whether the material helps students make connections between standards and their prerequisites.

***Indicators to measure the goal 1***



- A. The Material identifies and attends the prerequisite ideas or skills.
- B. The material makes connections between mathematical ideas and their prerequisites.
- C. The material provides scaffolded practice in each lesson that supports the development of targeted concepts and gradually formalize concept with verbal representation and mathematical model layered on it.
- D. The material identifies and explains misconceptions among students that are relevant to the topic.

***Goal 2. The material supports students to understand key conceptual ideas of mathematics which are useful in particular contexts.***

If students are given opportunities to formulate their own classifications for mathematical objects, and/or apply classification devised by others then they learn to differentiate and recognize the properties of objects. They also improve mathematical language and definitions. Students demonstrate “*conceptual understanding*” in mathematics when they recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulative, and varied representations of concepts (NAEP, 2003). However, Driscoll (1999) argue that it is not enough for student to merely be able to use multiple representations. Students must comprehend the connections among multiple representations, and how those representations relate to one another. For this, students compare different methods for doing a problem, organize solutions and/or identify the reasons of errors in the solutions. They begin to recognize that there are different trails through a problem, and develop their own chains of reasoning. Responding to this goal involves examining, whether the material includes pictorial models, diagrams, graphs, images and representations for important concepts and skills. Whether these representations and models are extended from prior grades to the next grades? These multiple models which representing mathematical ideas can help students make connections among the models and deepen conceptual understanding. Whether clear explanations of concepts and connections to other concepts indicated.

***Indicators to measure goal 2***

- A. The material includes conceptual problems and conceptual discussion questions.

- B. The material features opportunities to identify correspondences across Mathematical representations.
- C. The material uses pictorial models, diagrams, manipulatives for important concepts and skills and interrelate them with varied representation of concepts.
- D. The material includes application of facts and definitions and compares, contrast and integrate related concepts and principles

## 5.2. Procedural Fluency

Procedural fluency is a critical component of mathematical proficiency. Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently (Kilpatrick et al. 2001:121).

Procedural fluency refers to the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures (Kilpatrick et al. 2001:121). Before the promulgation of standards-based Mathematics curriculum, procedural fluency was given much emphasis in Mathematics curriculum materials at the cost of ignoring conceptual understanding and application (Hiebert, 1999). Students need opportunities and experience to integrate concepts and procedures. When students are engaged to use their own strategies and procedures in solving problems then they develop procedural fluency. Students use and justify formal procedures as well as commonly used methods if they are engage in an appropriate experience. They can also support and justify their choice of procedure. Procedural fluency and conceptual understanding are interwoven. Conceptual understanding makes procedural skills easier, less likely to common errors. On the other hand, an appropriate level of procedural fluency is needed to learn many mathematical concepts with understanding, and procedural skills can help build up that understanding (Kilpatrick et al. 2001:122). A sufficient level of skill practice is necessary for students to develop competency in other strands of proficiency.

***Goal 3. The material presents variety of mathematical procedures based on conceptual understanding, to build on fluencies and procedural skill in students.***

Skill practice should be specific, engaging, focused, and distributed (Rohrer, 2009). Plentiful practice can be unproductive or create math anxiety (Isaacs, Andrew & Carroll, 1999). Likewise

worked examples can perform as a key instructional tool, allowing teachers to understand how students analyze why procedures work or don't work and consider what procedure might be most appropriate in a given situation (Booth, Lange, Koedinger, & Newton, 2013). So the material explicitly makes it clear that why, when and how a procedure is applied. There must be a coherent sequencing while developing procedure. Whether materials help students make steady progress throughout the year toward fluent computation? Manipulatives and concrete representations such as diagrams that boost conceptual understanding are connected to the written and symbolic methods.

### ***Indicators to measure the goal 3***

- A. Materials include appropriate number of problems and worked examples based on procedural skills and interwoven with students developing conceptual understanding.
- B. Material applies accurate, flexible and coherent procedures to transform procedures to different problems and contexts.
- C. Material encourages students in developing and extending their own procedures using previously learned procedures to integrate concepts and procedures.
- D. The material explicitly informs teachers to engage students to identify and justify the appropriate procedures by comparing familiar and formal procedures.

### **5.3. Strategic Competense**

According to (kilpatrick et al. 2001), strategic competence means the students' capability to formulate mathematical problems, how to represent them, and solve them. The teacher should provide variety of problem solving techniques to students so that they should be able to formulate the problems of different situations out of school. The teacher should develop strategies for problem solving and generate structures. Using these structures students can apply their own problem solving strategies. Consequently, students will overcome the feelings of anxiety and become successful problem solvers. Polya (1957) presented 4-step problem solving model: understand the problem; devise a plan; carry out the plan; and look back at the solution. In *How to Solve It*, Polya explained powerful problem solving strategies such as making generalizations, making use of analogy, re-formulating a problem, manipulating the solution of related problems, exploiting symmetry, and look back at the solution. This model has been

reviewed by many researchers and has experienced many revisions. The National Council of Teachers of Mathematics (NCTM) included problem solving in NCTM's Principles and Standards for School Mathematics (2000). The National Mathematics Advisory Panel (NMAP, 2008) found that explicit instruction in problem solving is suitable for students, especially students who may struggle with learning mathematics. Explicit instruction is also appropriate for struggling students. Gersten, et al., (2009) analyzed six studies in which explicit instruction was used with students with special needs and found that "explicit instruction can expressively develop proficiency in solving word problems and operations across grade levels and students of multiple learning needs." (p. 21).

Mental representations of problems, developing mathematical relationships and making innovative solution strategies are also key for students to become proficient problem solvers. Kilpatrick et al. (2001) referred flexibility as the fundamental characteristic for problem-solving process. Contrary to the routine problems, non-routine problems require flexibility because students do not have prior workable solution methods for non-routine problems based on their previous practice. Learners need productive thinking to formulate and solve non-routine problems (p.126).

***Goal 4. The material provides explorative tasks and guide teachers to engage students in exploring mathematical investigation.***

This goal here is to examine whether the material guide teachers to assist students to formulate and construct their own strategies in solving problems. Ineffective strategy choices may result in less acquisition of higher order thinking and mental computation (Wu, 1999). Thus, explicit instruction can work to assist students to select more appropriate and effective strategies. Does the material include problem solving lessons throughout in each content domain? Do students have the opportunity to apply problem solving skills in real life situations and problem-solving settings? Are scaffolded models including sequence of example problems provided before students are asked to solve problems on their own? Does the material contain exploratory questions that help students make sense of their experiences?

***Indicators to measure the goal 4***

- A. Materials provide guided prompts to make students engage and get initial mastery about mathematical investigation.
- B. The material explicitly helps and instructs teachers to create an inquiring environment in the classroom where questions are answered through mathematical investigations.
- C. The material encourages students to use alternate strategies in designing investigations to gather evidence in response to questions.
- D. The material contains problems of variety of complexity and encourages students to make investigations independently based on their previous learning to seek solutions of problems.

***Goal 5. The materials encourage students to explain/communicate their investigations and mathematical thinking orally and in writing with others.***

Does the material encourage students to develop explanations using their common experiences and findings from the lessons in mathematical investigations? Whether the material includes such problems that help students express their thinking and explanations? Does the material include effective questions which assist teachers enquire from students: (1) what pattern(s) did you notice? (2) What evidence do you have for your claims? (3) How can you best explain/show your findings? (4) What are some other explanations for your findings? (Marshall et al. 2009). The material must provide opportunity for teachers to involve students compare their understanding with those of others so that they should reflect and review their ideas if needed. Whether the material assist teacher for alternative justifications after students explains their ideas?

***Indicators to measure the goal 5***

- A. The material prompts students to explain the procedures of mathematical investigation at hand.
- B. The material encourages students to explain, clarify and represent concepts.
- C. The material engages students in meaningful discussions and interactions in such a way that they communicate their understanding with peers and teachers and reflect and revise their ideas.
- D. The material guide teachers in the consideration of alternative explanations.

***Goal 6. Materials make connections within the subject and in contexts outside mathematics to develop strong mathematical understanding.***

The material should focus on strengthening conceptual connections between new and previous experiences. Students should be engaged and encouraged to use learning to explain new ideas. Assist teachers to probe into students' explanations of mathematical investigations that help students draw reasonable conclusions from evidence and data. Small group discussions and cooperative learning experiences may help students express their understanding of the subject. To respond this goal also involve examining whether the material provide an opportunity to engage students in new situations and problems that require the application of identical or similar explanations and generalize the concepts, processes, and skills.

***Indicators to meet the criteria***

- A.** The material provides and assists teachers engage students in making conceptual connections between new and previous learning experiences.
- B.** The material includes non-routine problems and contextual tasks that encourage students to formulate, model and apply their understanding.
- C.** The materials engage students in analyzing evidence from the data arising from their investigations and draw reasonable conclusions from evidence and data.

**5.4. Adaptive Reasoning**

NCTM Standards for School Mathematics (2000) describes reasoning in the following way: *Instructional programs from prekindergarten through grade 12 should enable all students to: recognize reasoning and proof as fundamental aspects of mathematics; make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs; select and use various types of reasoning and methods of proof.*

According to Kilpatrick et al., (2001), adaptive reasoning' means the "capacity for logical thought, explanation, and how to justify conclusion". Kilpatrick's view about adaptive reasoning is not limited to the formal proof and deductive reasoning; rather adaptive reasoning encompasses intuitive and inductive reasoning based on pattern, metaphor and analogical correspondence (p.129). Mathematics is not just a collection of rules and formulas to apply;

rather there is strong reasoning behind the rules and construction of formulas. Adaptive reasoning as a centrality to the mathematics learning has widely been argued (Powell, Francisco & Maher, 2003; Stigler & Hiebert, 1997). According to Kilpatrick, adaptive reasoning is the glue that holds everything together (p. 129). Alexander, P. A., White, C. S., & Daugherty, M. (1997) identified three conditions for young children who demonstrate good reasoning skill: They have deep understanding of knowledge, the problem is comprehensible and interesting, and the situation is familiar and encouraging.

One of the strong mechanisms to demonstrate sophisticated reasoning abilities is to make analogical correspondences and mental representations (English, L. D. 1997a). Through the variety of representation-building practices, learners can exhibit sophisticated reasoning abilities. Also analytical skills of students can be promoted with the help of analogies. Students should be well acquainted with specialized vocabulary as well as the relationship such as contrasting, comparing and sequencing to understand analogies. Famous researchers have claimed that analogical reasoning may be central to learning of abstract ideas (e.g., Brown & Kane, 1988), procedures (Ross, 1987), novel mathematics and how to transfer mental representations across situations (Novick & Holyoak, 1991). Novick & Holyoak, (1991) has also found that analogical comparison can result in construction of abstract representations to symbolize the underlying structure of source and target objects, thus increasing reasoning capacity to transfer learning across situations.

Kilpatrick commented that the Justification and proof are hallmark of formal mathematics. He pointed out that students should be given opportunities to learn to justify their mathematical concepts across grade levels because the development of proficiency needs much practice and may occur in a long period of time (p.130). Several researchers have studied the nature of proof, justification and explanation in school mathematics textbooks. A study conducted by Stylianides (2008) investigated how proof is supported by standards-based curriculum for middle grades. The researcher identified that about 5% of student tasks involved proof in Harel and Sowder's (1998) broad sense of proof schemes. The findings also suggested the need to enhance learners' understanding of legitimate mathematical proof. Stacey, K & Vincent, J (2009) examined the reasoning in the explanatory text in nine Australian eighth-grade textbooks those introduced new

mathematical rules or relationships. It was seen that most textbooks presented explanations for most areas rather than providing "rules without reasons".

According to Kilpatrick et al, (2001), adaptive reasoning is interconnected with the other strands of proficiency, particularly during problem solving. Learners use heuristic approaches and strategic competence to formulate and represent a contextual problem that may lead them to devise solution strategy of the problem. To justify this proposed strategy learners use adaptive reasoning (p.130). He further explained that conceptual understanding provides metaphors and representations that can serve as a source of adaptive reasoning, which, taking into account the limitations of the representations, learners use to determine whether a solution is justifiable and then to justify it (p.31).

***Goal 7. The material provides opportunities for logical thinking about the relationships among concepts and situations.***

Materials should provide appropriate opportunities for students to think logically and mathematically. The goal here is to examine whether the interactional activities provided by instructional materials are consistently dialogical, engaging, and motivating across topics and grade levels? Through conversational interactions, argumentative discussions and reflections, students provide evidence of connections and representations to personal applications. Students also critique the responses and arguments of peer students and the teacher. The material should encourage students to solve problems in more than one way, allow students to develop their own approaches, encourage collaboration between students. Students are more likely to reason when they are provided opportunities to develop their own solution strategies, connections and justify conclusions than if they are provided with repeating arguments. The material should include grade appropriate tasks whose solution is not known in advance. Thus materials develop reasonable growth of students' mathematical reasoning and specialized language from early grades up through high grades.

***Indicators to measure goal 7***

A. The material explicitly integrates the new strategies with familiarized strategies and makes meaningful connections between representations and strategies.



- B.** The material includes and instructs teachers to provide students with concrete referents such as objects, illustrations, diagrams, and actions to help students construct arguments and evaluate mathematical proofs.
- C.** The material consists of mathematically rich investigative tasks and encourages students to use their own strategies to infer and justify their conclusions.
- D.** The material stimulates students to construct reasonable arguments and critique the arguments of peers and teachers.

### **5.5. Mathematics Disposition**

Productive disposition is defined as “the ability to make sense in mathematics, to perceive mathematics as useful and worthwhile, to believe that consistent effort in learning mathematics gives good result, and trust on oneself as an effective learner of mathematics” (Kilpatrick et al., 2001, p: 131). Middleton, Leavy, Leader & Valdosta (2013) found that “with curriculum intended to emphasize utility and interest, students showed a high degree of motivation”. Also, their achievement increased intensively, in part, as the effect of this increased motivation. In the classroom situation self-efficacy refers to the routine environment of the classroom, including boosting the learning and doing of mathematics (Boaler, 2002), encouraging socio-mathematical norms (Yackel& Cobb, 1996). The capability to sense in mathematics is related to how students believe the nature of mathematics. That is whether the mathematics is a challenging or an intelligible subject, whether mathematics is based purely on rules or a useful subject in real life. One of reasons why students are not likely to see sense in mathematics is belief that mathematics is comprised purely on arbitrary rules and memorization of procedural steps, and there is no use of these rules in real life situations. A research conducted by National Assessment of Educational Progress (NAEP) to check the belief of students about the nature of mathematics. The research found that 40% of eighth-graders believed that mathematics learning is just memorizing rules and follow certain procedural steps (NRC, 2001). On the contrary, students having strong productive disposition believe that mathematics should make sense. They are confident in their knowledge and ability and know that when and how they are making sense in mathematics.

Kilpatrick argued that mathematics is comprehensible and with diligent effort students can be proficient at other strands of mathematical proficiency. For the development of productive

disposition other strands needs to be developed (p.131). The more students will be proficient in conceptual understanding and strategic competence, the more sensible mathematics becomes. In turn students' beliefs about learning of mathematics become encouraging. Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics (Kilpatrick et al, 2001:131).

Self-efficacy is another component highlighted by Kilpatrick et al to develop productive disposition. Students' self-efficacy has a direct relation with the amount of effort a learner is making on a mathematical task. Students with high self-efficacies in mathematics consider themselves as efficient in doing mathematics and perform well on mathematical tasks. Pajares (1996) highlighted that people with low self-efficacies are of the view that particular kinds of mathematical tasks are too difficult for them to solve. They give up to put any kind of effort on such problems because they think that the efforts would be in vain. This pessimistic belief can in turn lead to mathematics anxiety and feelings that mathematical problems are more difficult than they really are. Studies also show that efficient teaching strategies and encouraging learning environments in the classroom play a central role in the development of intrinsic motivation which in turn helps students develop mathematical dispositions (Cobb, Wood, Yackel, & Perlwitz, 1992; Middleton, 1995; Middleton & Spanias, 1999). Intrinsically motivated students are more confident in doing mathematical tasks, and are persistent in using more challenging strategies to find the solution of contextual tasks (Lepper, 1988). Thus intrinsic motivation is correlated with self-efficacy and positive dispositions towards mathematics understanding (Maher, Yankelewitz, & Mueller, 2010).

***Goal 8. Material provides a challenging but purposeful and conducive learning opportunities for teachers and students.***

It can be deduced from the above literature that the instructional resources teachers use to teach mathematics should contain an understanding and awareness of students' mathematics dispositions. The material includes such content which helps develop students' identity formation and development. Responding this instructional goal also requires examining whether the material integrate multiple teaching methods highlighting contextualization and real life

application? Whether material offers effective and learner-centered teaching methods requiring active student participation which include problem solving activities with multiple ways of representations, inquiry-based learning approaches and opportunities to develop links between real world situations and mathematics? Teachers should encouraged students to explain their understandings of the task, and value and respect their ideas. This respect prompts learners become intrinsically motivated to succeed at mathematics and this positive behavior inculcate ascending dispositions towards mathematics. Students become independent learners and enjoy doing and sharing mathematical ideas. The material should also support and provide guidelines for teachers to develop supportive and encouraging mathematical environment, engaging and collaborative tasks to develop mathematics disposition.

### ***Indicators to measure the goal 8***

- A.** The material provides appropriate type of scaffolding strategies up to a certain level to enable students demonstrate their understanding independently.
- B.** The material explicitly provides guideline for teachers to create engaging and encouraging learning environment in the class, and value and respect students' ideas.
- C.** The material integrates multiple teaching methods requiring active students' participation and provides links between real world situations and mathematics.
- D.** The material recommends authentic links for online materials for supporting teachers and students understanding about a specific skill or a particular idea and its application.

### **5.6. Assessment**

Assessment is an integral part of effective teaching and learning (NCTM, 2000, P.22). Assessment activities provide students and teachers with opportunity to monitor their own understanding on the key ideas of mathematics proficiency (Kilpatrick et al, 2001). Textbooks and other instructional materials must incorporate good assessments including both formative and summative assessment strategies to measure progress of teaching and learning of teachers and students. In the Assessment Standards for school Mathematics, assessment is described as *“the process of gathering evidence about a student’s knowledge of, ability to use, and dispositions toward, mathematics and making inferences from that evidence for a variety of purposes”* (NCTM, 1995, p. 3). Assessment is not just a collection of mathematical tasks to

practice rather its aim is to provide and improve students' in-depth understanding and a sustainable proficiency in mathematics. Assessment measures what is taught and what gets learnt. This means that Assessment should focus on what students already learned, as well as on the areas for need improvement. Assessment is related to teaching and learning because, what is assessed becomes what is taught, the assessment method determines the teaching approach, and different content is suited to different assessment methods (QSA, 2013). For Educators to make important instructional decisions, purposeful assessment strategies can be used effectively (Romagnano, 2001; Wiliam, 2007). In addition, it helps teachers to find methods to develop mathematical proficiency (NCTM, 2000). According to (Kilpatrick et al 2001), assessment should support the development of students' mathematical proficiency. Such assessments help teachers measure the actual learning and revise their classroom instruction to enhance learning (p. 444). The form of expressions that occur in the material should be reflected in assessment. In addition, students of all diversity levels should be given opportunities to demonstrate what they have acquired from the variety of activities incorporated in the materials (Klinger et al. 2015).

Assessments must focus on the variety of skills, applications, and representation that divulge what students are expected to learn (project2061). Further, just giving a list of exercise questions at the end of a unit is not enough rather it requires formative assessment tasks to be incorporated throughout instructional material. Appropriate attention should be given on such tasks to assess curriculum standards and multifaceted forms of mathematics. NCTM Assessment Standards for School Mathematics proposed three criteria as evidence to enable instructors “(1) Assessment should examine the effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions; (2) Assessment should make instruction more responsive to students' needs; and (3) to ensure that every student is gaining mathematical power” (NCTM, p. 45). Instructional materials must include assessments that lead teachers engage students in mathematical investigations, making predictions and analyzing and justifying the results. Students should be given consistent opportunities to reflect about their work.

***Goal 9. The material provides variety of assessment opportunities to monitor and support students' progress in Mathematics.***

***Indicators to meet the criteria***

- A. Material attends sufficient number of models and examples of wide range of complexity to accommodate variety of needs of students and refine students' understanding of the important mathematical concepts.
- B. The material includes variety of formative Assessment activities (e.g. diagnostic tests, Model-eliciting activities, generative activities, group discussions, constructive quizzes, peer assessment tasks, etc.) in order to modify teaching and learning and improve student attainment.
- C. Materials guide teachers to offer effective and descriptive feedback on students' work to enhance students' motivation towards achievement goals.
- D. Material provides and guides teachers to develop reflective and self-assessment activities for students to make them actively involved in the process of learning and enable them evaluate their own leaning.

## **6. Discussions and Implications**

The standards-based curriculum is currently gaining its roots in educational realm and several models and tools are currently exist (e.g., project 2061, CCSS Mathematics Curriculum Materials Analysis Project etc.) to evaluate standards-based curriculum materials. However, it is a significant concern among researchers that mathematics curriculum is evolving (e.g. BerinderjeetKaur, 2014; Gordon, Sheldon P, 2013) and hence need an advanced and evolved framework to capture emerging feature of curriculum materials which are necessary to develop students' mathematical proficiency. Thus the proposed framework is expected to present a varied kind of mechanism that allow textbook publishers, teachers or curriculum developers to look deeply into the critical features of their instructional materials before drafting or selecting materials, to make sure the meaningful teaching and learning should take place. If it is necessary for curriculum materials to focus mathematical proficiency, then it must explicitly incorporate all five strands of mathematical proficiency discussed above. Further, it is also significant to know how to measure mathematical proficiency i.e. effective formative Assessment strategies are critical to enhance meaningful and dynamic learning proficiency in all strands of mathematics (Keeley et al. 2005; NRC 1996). Also authentic formative assessments activities help facilitate more up-to-date and deep instructional practice. The given framework provide an evolved indicators to analyze that at what extend published instructional materials support and integrates mathematical proficiency. It is expected that teachers and reviewers will acquire deeper

evaluation procedure skills while being involved in the evaluation process experience, and consequently develop deep conceptual understanding of mathematics. The framework is expected to provide a significant insight to textbooks' authors and publishers to take into account, the important and critical factors in designing curriculum materials. This framework may also work as a monitoring tool for governments' monitoring teams to measure whether certain published materials are aligned with the national curriculum.

## 7. Conclusion

The conceptual understanding, procedural fluency, adaptive reasoning, strategic competence and mathematics disposition strands, which form the core principles, of mathematical proficiency, are necessary for strong understanding of mathematics teaching and learning. However, by explicitly and efficiently integrating authentic Assessment strategies in curriculum materials and teaching process, mathematical proficiency can be enhanced more deeply. The proposed framework focuses on the extent to which standards for mathematical proficiency are incorporated and integrated in instructional materials. It synchronizes proficiency strands and Assessment strategies into one balanced framework that provides reviewers with a vigorous mechanism to analyze instructional materials to improve teaching and learning. Although this research does not claim to encompass all needs of instructional materials evaluation in mathematics curriculum across the world but it is justifiable to say that the current research about the framework of instructional material evaluation is a significant development and evolution over the previous published frameworks.

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